Heavy Quarks and the Bulk Quark Gluon Plasma

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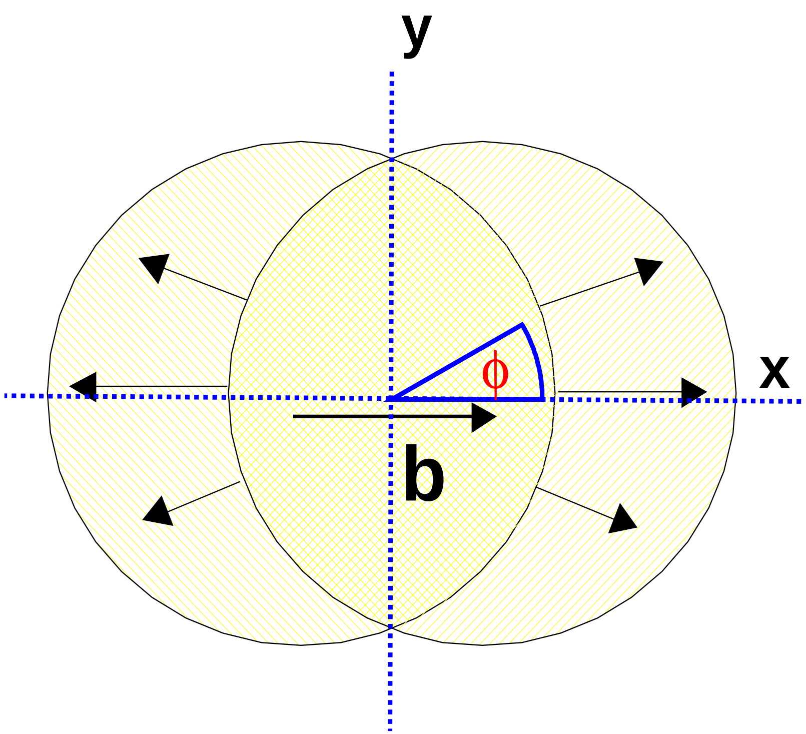
Motivation and Recap

Observation:



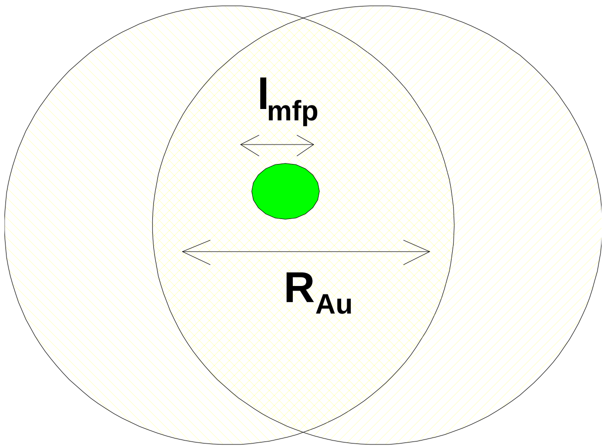
There is a large momentum anisotropy:

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \approx 20\%$$

Interpretation

- The medium responds as a fluid to differences in X and Y pressure gradients

Hydrodynamics:



- For hydrodynamics need:

$$\frac{\ell_{\rm mfp}}{R_{\rm Au}} \ll 1$$

- How to define $\ell_{\rm mfp}$?

$$\ell_{\rm mfp} \sim \frac{\eta}{e+p} \qquad e+p = sT$$

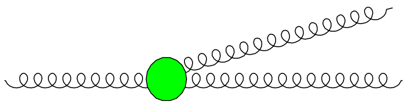
Condition:

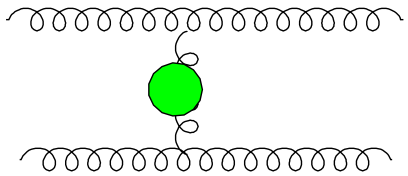
$$\underbrace{\frac{\eta}{s}}_{\text{Medium Property } \sim 1/\alpha_s^2} \times \underbrace{\frac{1}{RT}}_{\text{Experimental Property } \sim 1/2} \ll 1$$

Need $\eta/s \lesssim 0.3$ to have hydro at RHIC

What does $\eta/s < 0.4$ mean theoretically?

- Perturbation theory: (Baym and Pethick. Arnold, Moore, Yaffe)
  - Kinetic theory of quarks and gluons + soft gauge fields + collinear emission





$$\frac{\eta}{s} \simeq 0.3 \left(\frac{0.5}{\alpha_s} \right)^2$$

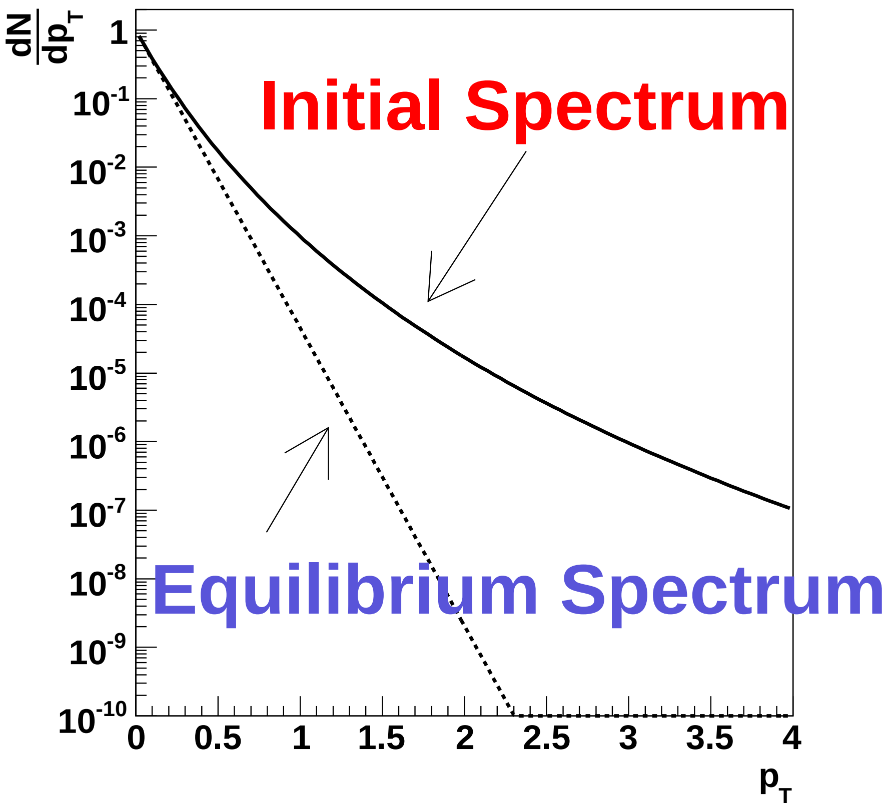
- $\mathcal{N} = 4$ Super Yang Mills at strong coupling (Kovtun, Son, Starinets, Policastro)
  - No quasi-particles.

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad \Longrightarrow \qquad \text{Conjectured Lower Bound}$$

The experimental results are within a factor of a few of the KSS bound

Check with other measurements!

Energy Loss of Fast Partons – Cartoon



- Power law initial spectrum:

$$\frac{dN}{dp_T} \propto \left(\frac{1}{p_T}\right)^{10}$$

- Exponential equilib. spectrum:

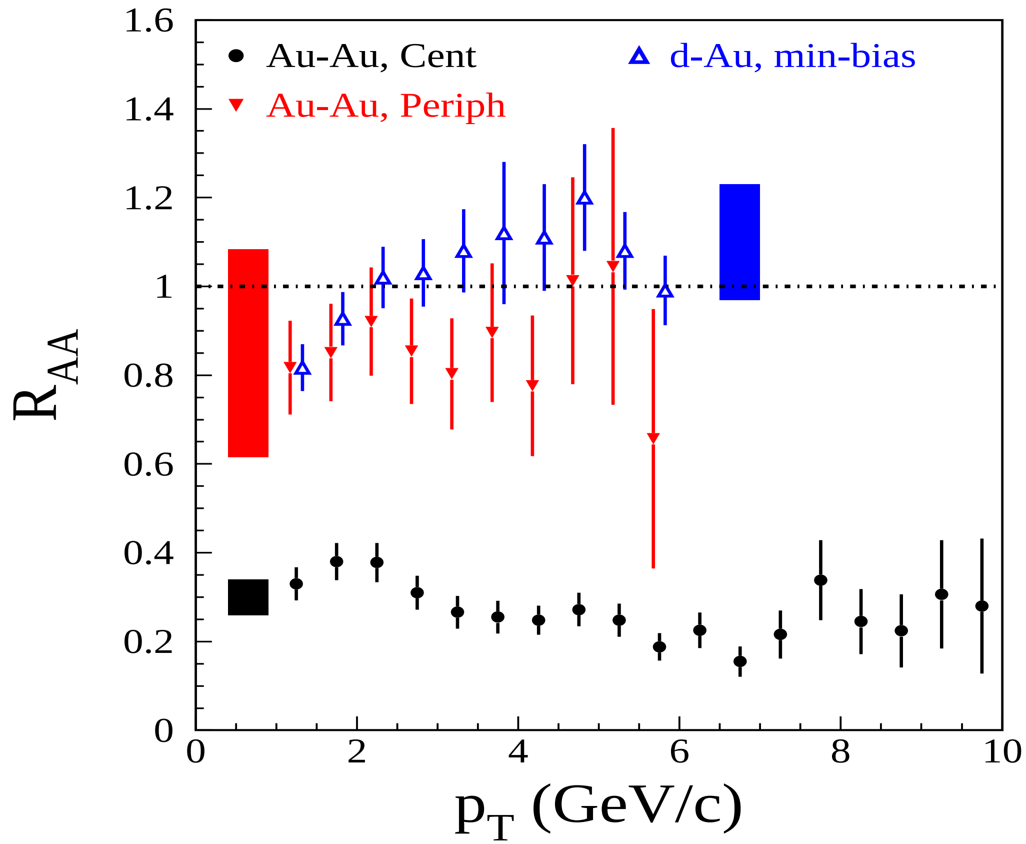
$$\frac{dN}{dp_T} \propto e^{-\frac{p_T}{T}}$$

The initial spectrum will lose energy and approach the equilibrium spectrum

Tells something about density and interaction rates

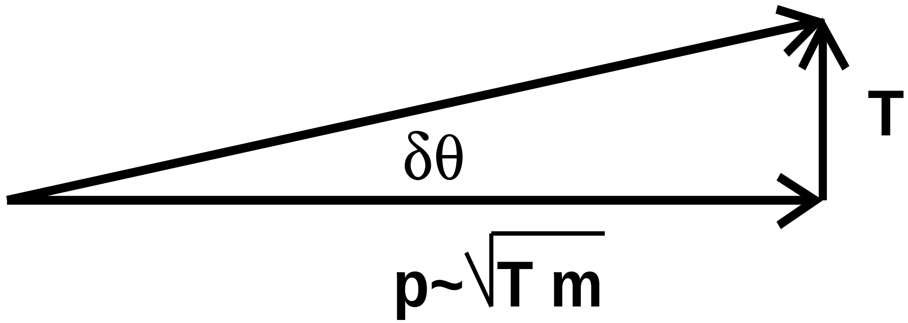
Data on π^0 p_T spectrum

$$R_{AA} \equiv \frac{\left(\frac{dN}{p_T dp_T}\right)_{\text{In AuAu}}}{N_{\text{coll}} \left(\frac{dN}{p_T dp_T}\right)_{\text{In pp}}}$$



Will the charm quark thermalize?

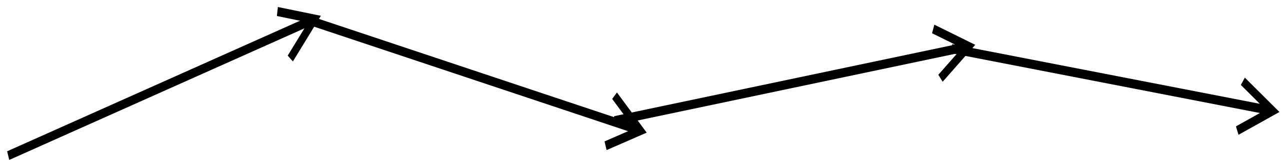
- In collaboration with Guy Moore



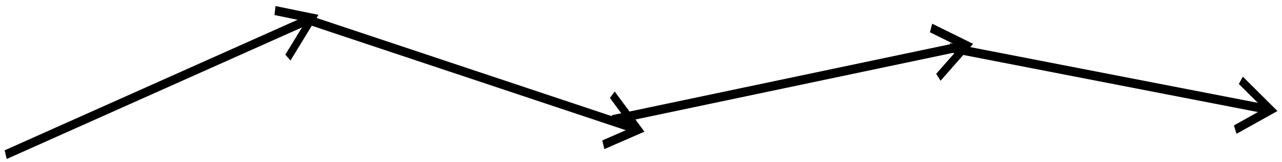
$$\delta\theta \sim \sqrt{\frac{T}{m}}$$

- The collision only scarcely changes the direction of the charm quark

- The charm quark undergoes a random walk suffering <u>many</u> collisions provided $\ell_{m.f.p} \ll L$



$$(\Delta\theta)^2 \sim N_{kick}\,(\delta\theta)^2 \sim N_{kick}\,\frac{T}{m}$$



$$(\Delta\theta)^2 \sim N_{kick}\,(\delta\theta)^2 \sim N_{kick}\,\frac{T}{m}$$

- For equilibration we need:

$$(\Delta\Theta)^2 \sim 1 \qquad \text{or} \qquad N_{\text{kick}} \sim \frac{M}{T}$$

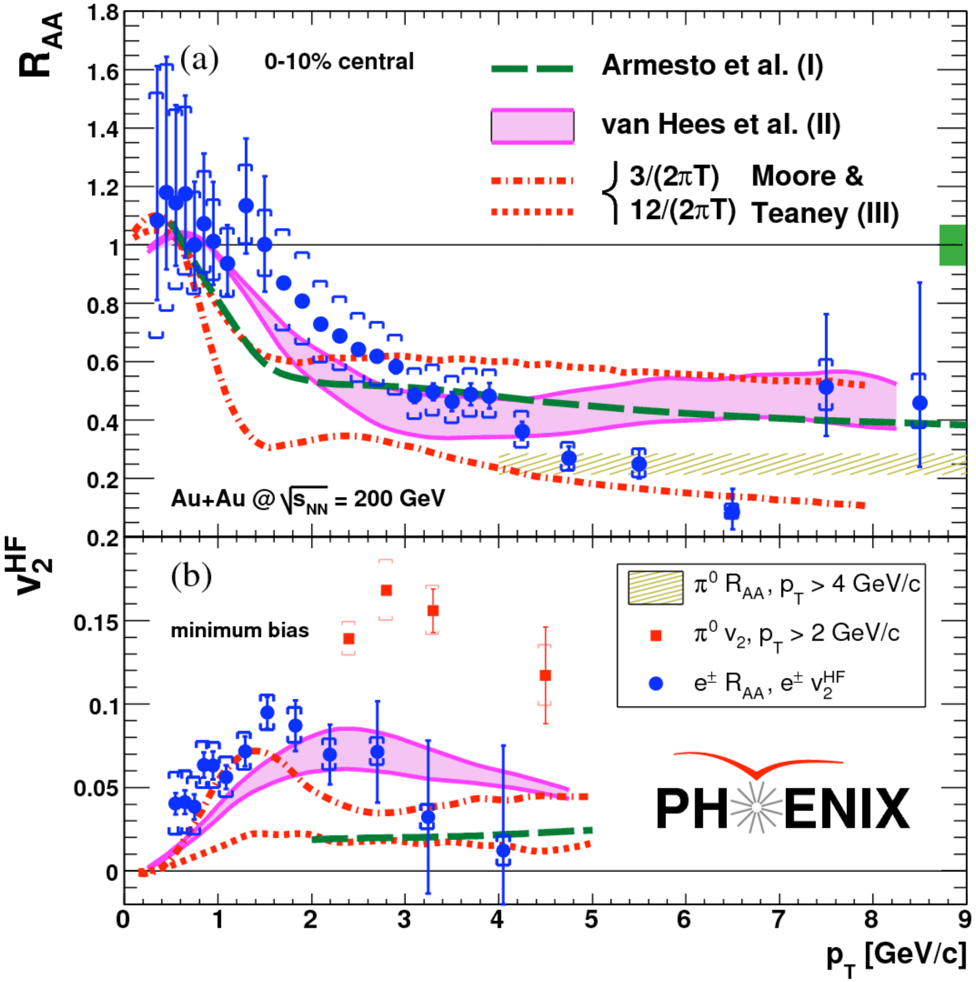
- Thus for charm equilibration we have:

$$\begin{aligned} \tau_R^{\text{charm}} &\sim \frac{M}{T}\tau_R^{\text{light}} \\ &\sim \frac{M}{T}\frac{\eta}{e+p} \end{aligned}$$

It takes a longer time to equilibrate charm.

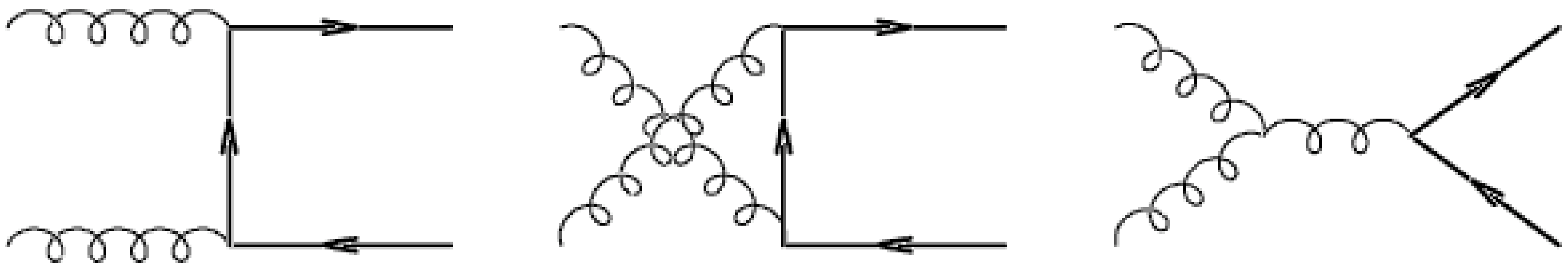
If you think you know η you should be able to compute the charm spectrum.

The goal of this lecture!

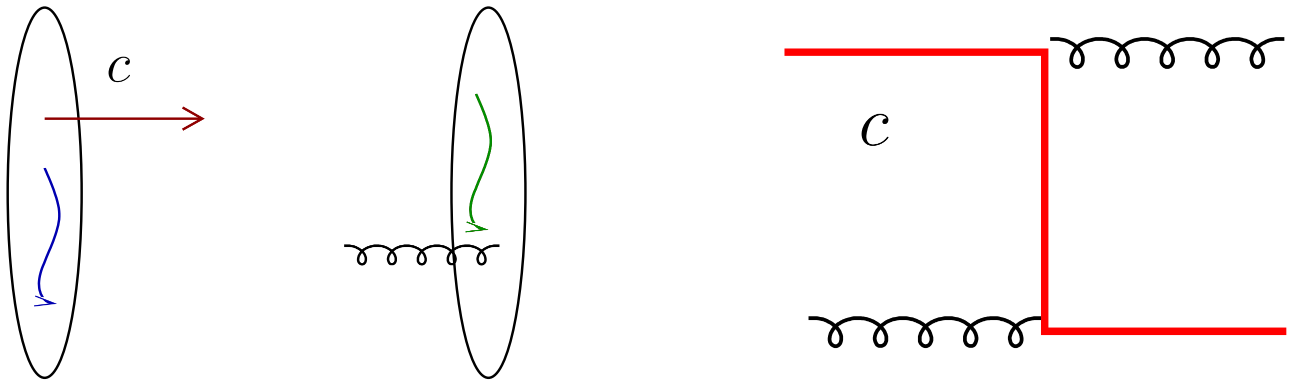


Heavy Quark Production in pp – (not really my business)

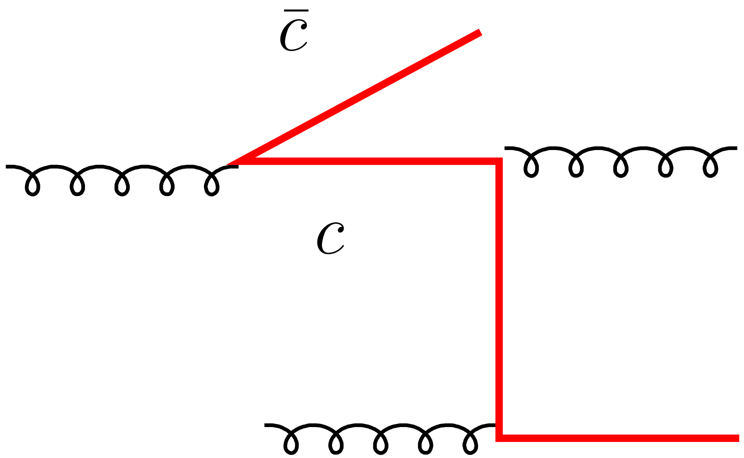
- Input heavy quarks from M. Cacciari, P. Nason, R. Vogt
- Quick review here based on talks by Matteo Cacciari
- Nothing to it – right?



A lot more to it actually:

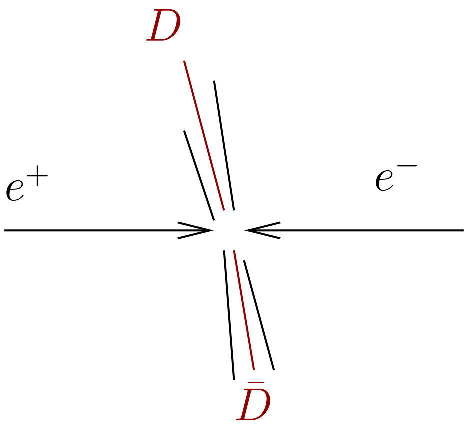


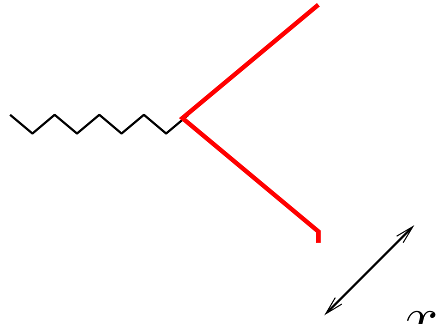
- A NLO calcuation gives the init. condits for the charm structure function



Starts to be dominant around RHIC energies

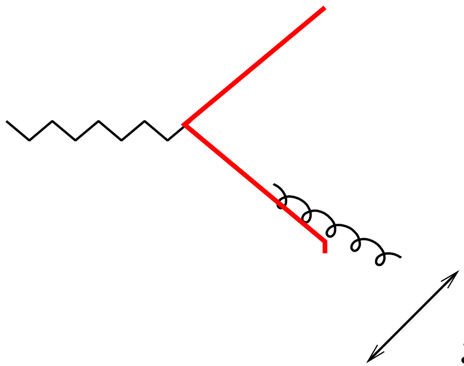
Fragmentation





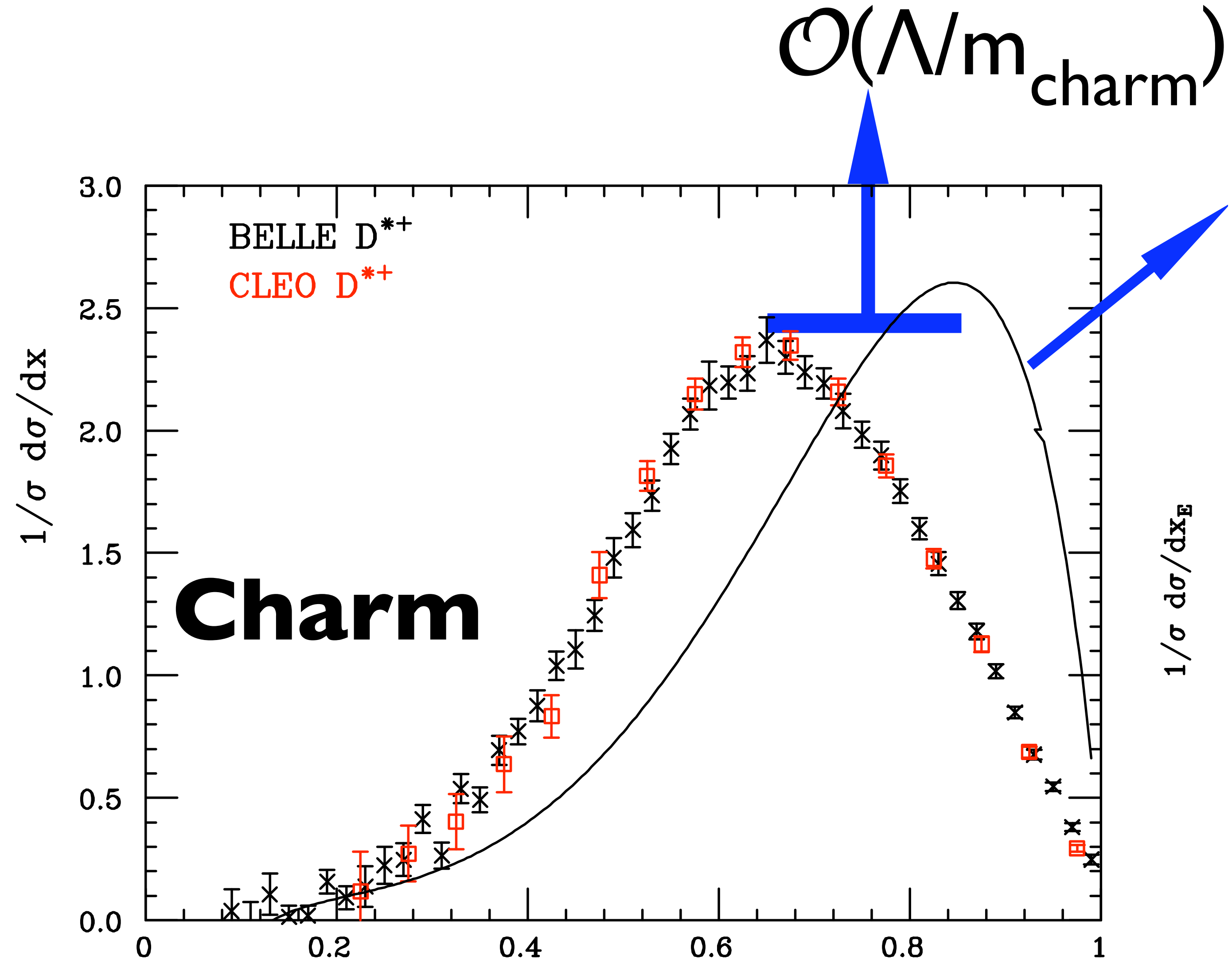
$$x = \frac{P_H}{P_{\text{jet}}}$$

- The D is born with a fraction of the quark momentum: $\frac{1}{\sigma}\frac{d\sigma}{dx} = D(x)$
- A NLO calcuation a heavy quark fragmentation function



$$x = \frac{P_H}{P_{\text{jet}}}$$

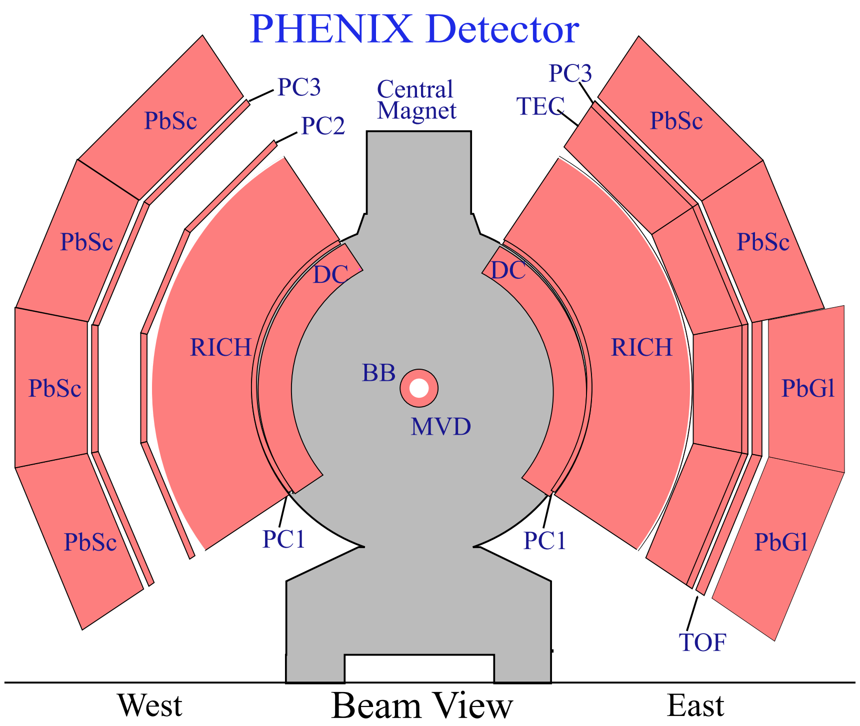
Hadronization of $c \to D$

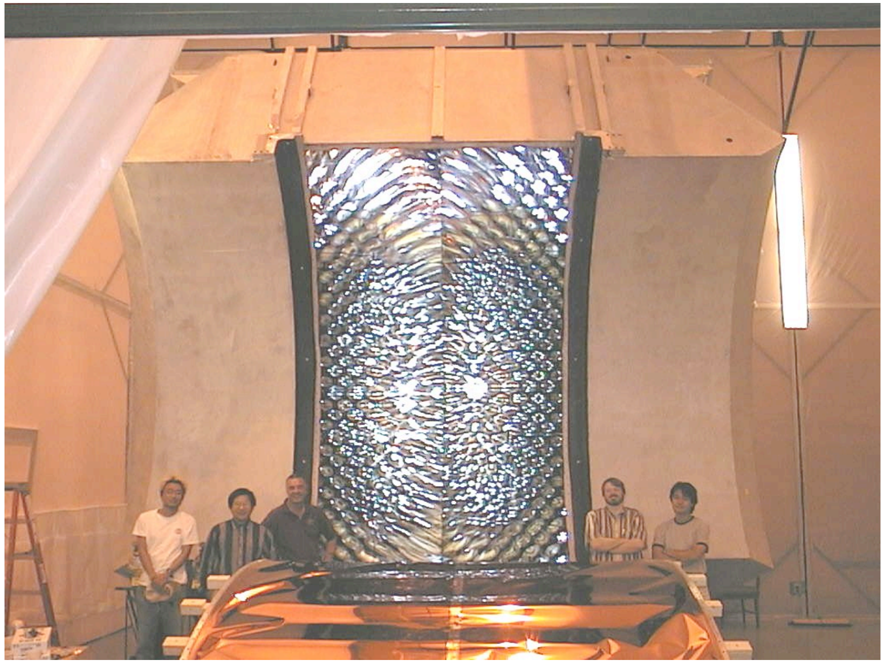


These fragmentation functions are very well known.

Finally decay into an electron: $D \rightarrow K^* e\nu$

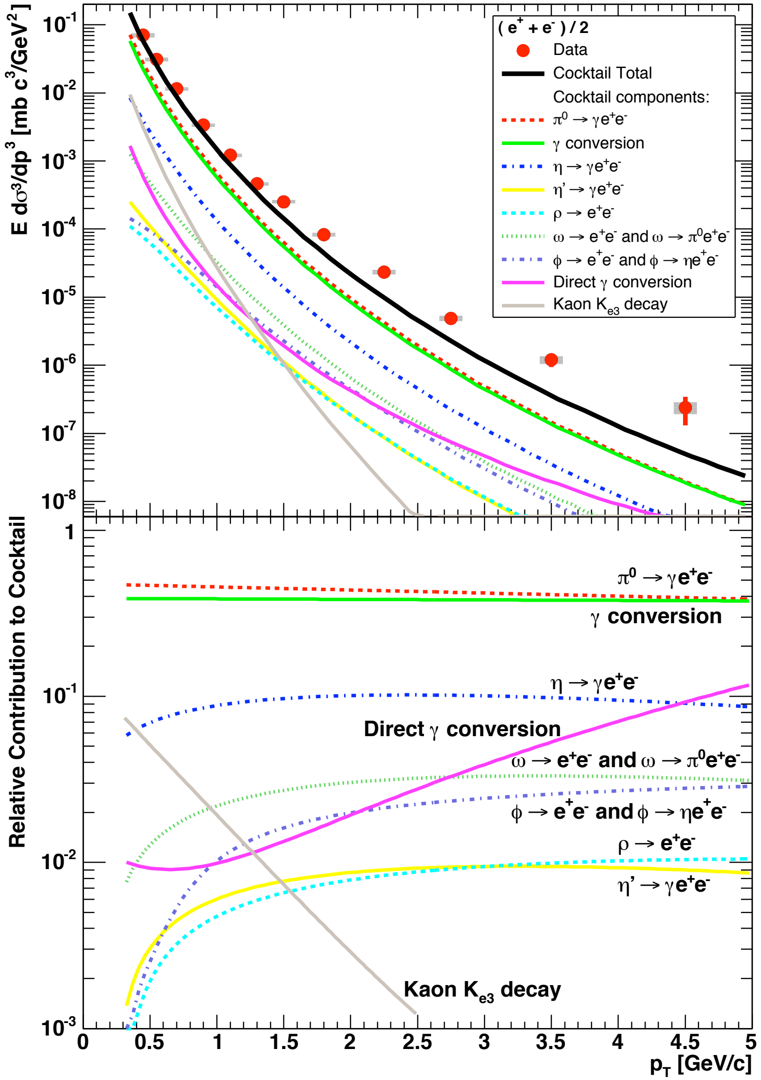
- The electron produces ringed light in the RICH





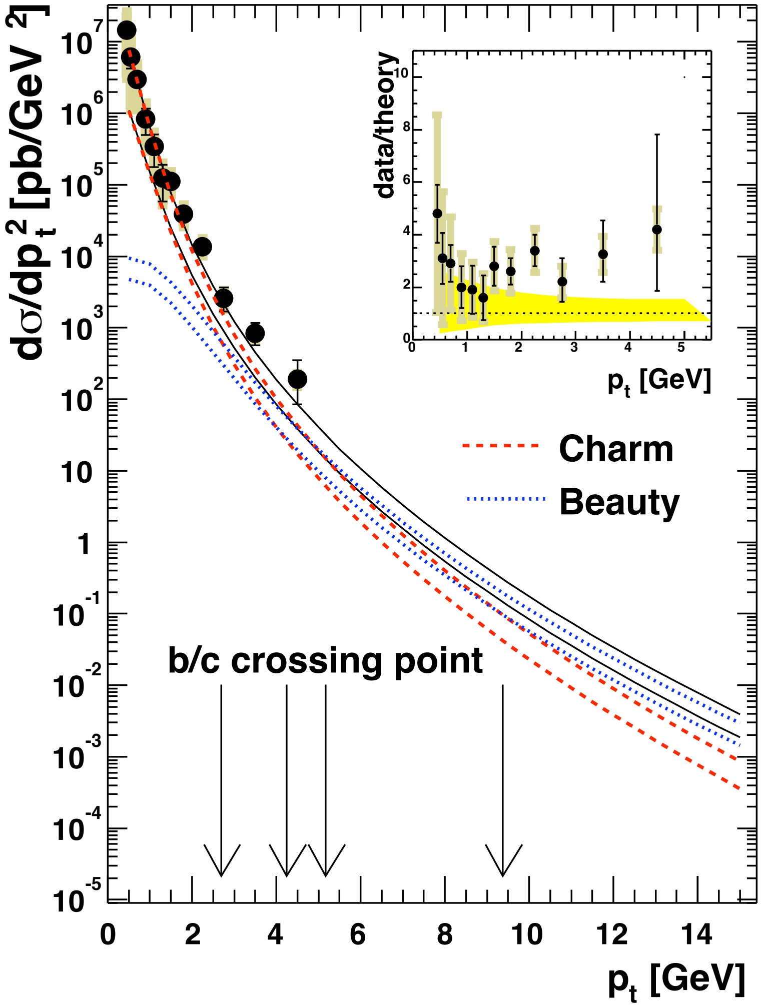
Measure the Cerenkov Ring w. Ring Imaging Cherenkov Detector

Measured electrons and comparison with FONLL

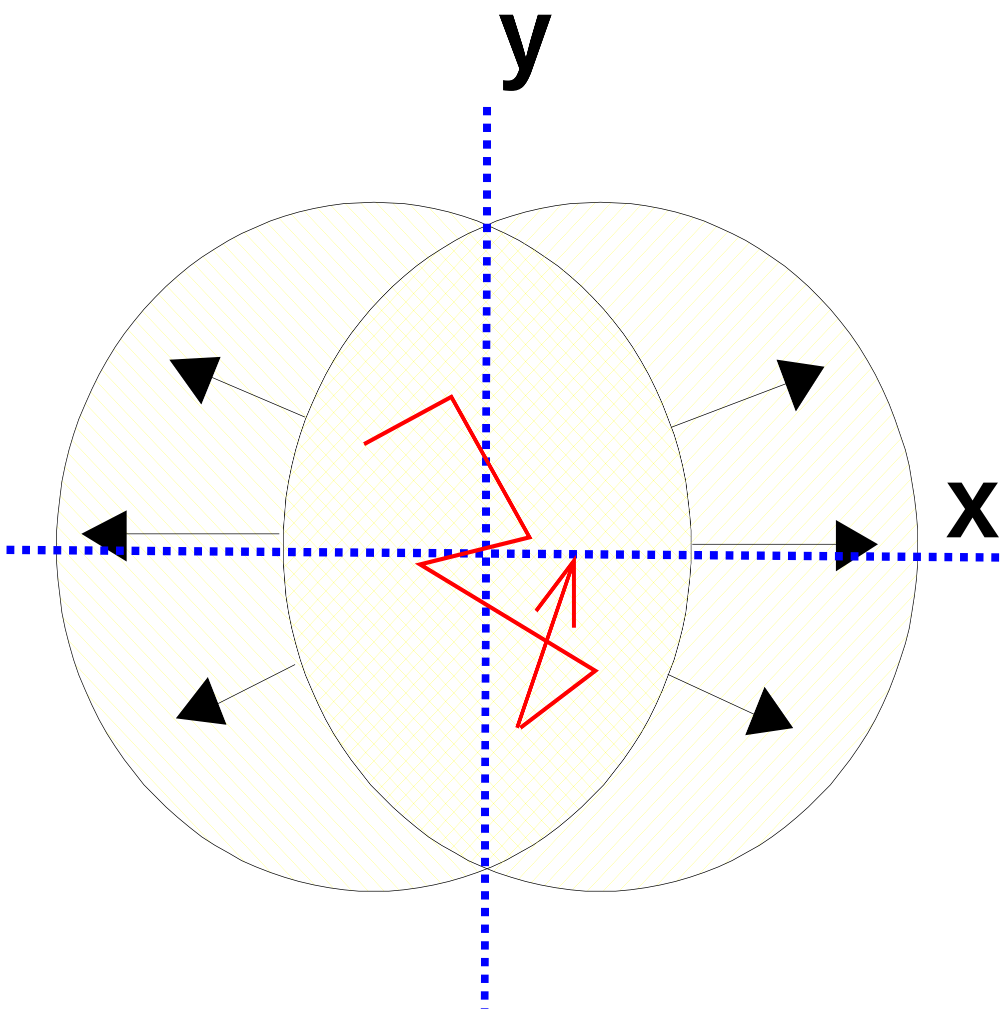


(a) p+p data [31]

⟹



Heavy Quarks



The heavy quarks will either relax to the thermal spectrum and show the same v_2 as all thermal particles or not depending on the typical relaxation times.

Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

$$\frac{dx}{dt} = \frac{p}{M}$$

$$\frac{dp}{dt} = -\underbrace{\eta_D p}_{\text{Drag}} + \underbrace{\xi(t)}_{\text{Random Force}}$$

- The drag and the random force are related

$$\langle \xi_i(t)\xi_j(t') \rangle = \frac{\kappa}{3}\delta_{ij}\,\delta(t-t') \qquad \eta_D = \frac{\kappa}{2MT}$$

$\kappa =$ Mean Squared Momentum Transfer per Time

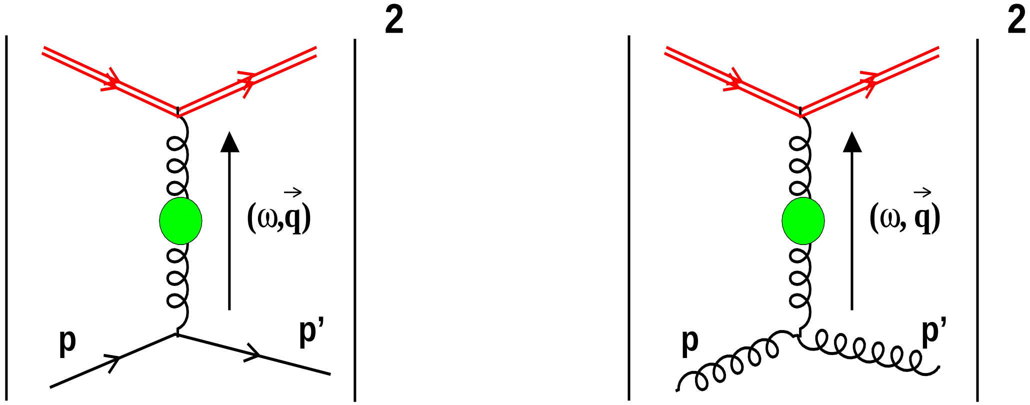
- Einstein related the diffusion coefficient to the mean squared momentum transfer

$$D = 2T^2/\kappa$$

All parameters are related to the heavy quark diffusion coefficient or κ

Computing κ in the perturbative QGP:

- κ is the mean squared momentum transfer per unit time:



$$\kappa = \int_{\mathbf{p},\mathbf{p}',\mathbf{q}} \mathbf{q}^2 \left[f(p)(1+f(p')) \left|M_{\text{glue}}\right|^2 + f(p)(1-f(p')) \left|M_{\text{quark}}\right|^2 \right]$$

- Radiation of the heavy quark line is suppressed by the velocity: $v^2 \sim \frac{T}{M}$

- See also Sevitsky, Braaten and Thoma

We did it

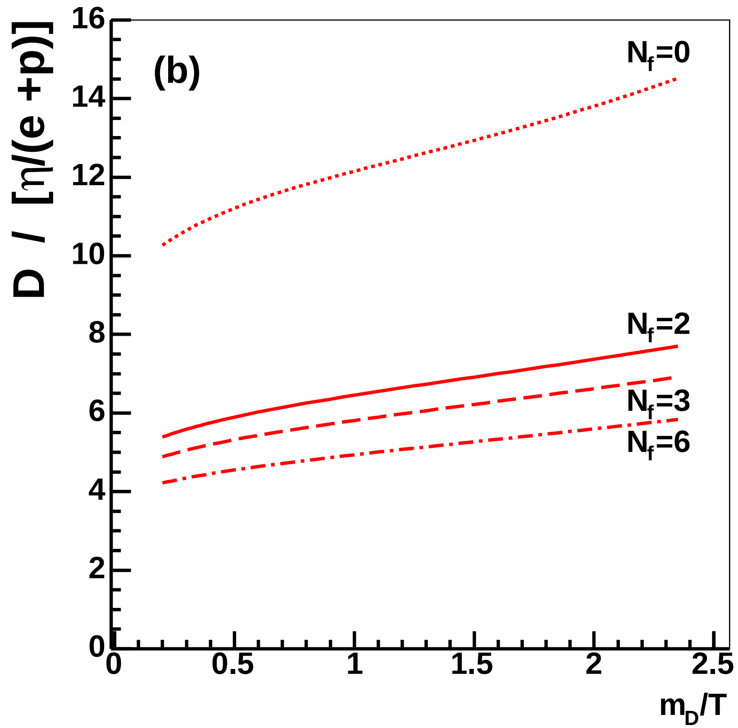
$$\kappa \sim T^3 \lambda^2 \log\left(\frac{T}{m_D}\right) \qquad \lambda = g^2 N$$

Perturbative estimate of the diffusion coefficient

$$D = \frac{2T^2}{\kappa} = \frac{36\pi}{C_F g^4 T}\left[N_c\left(\ln\frac{2T}{m_{\rm D}} + \frac{1}{2} - \gamma_{\rm E} + \frac{\zeta'(2)}{\zeta(2)}\right)\right]^{-1}$$

- Expectations:

$$\tau_R = \frac{1}{\eta_D} = \frac{M}{T}D \sim \frac{M}{T}\frac{\eta}{e+P} \qquad \text{So} \qquad D \sim \frac{\eta}{e+P}$$



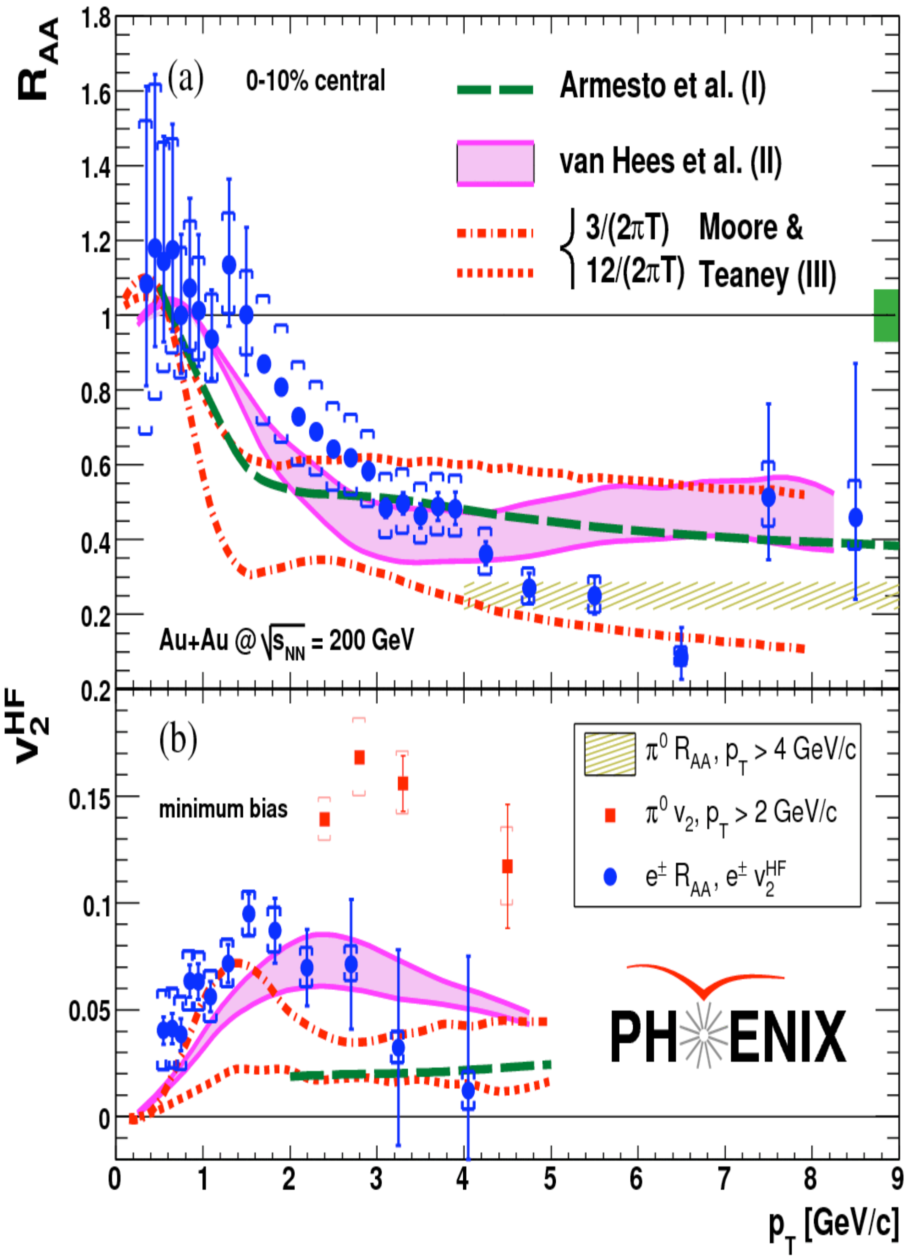
$$D \sim 6\frac{\eta}{e+P}$$

Application to Heavy Ion Collisions

- Generalize to mildly Relativistic quarks.
  - Assumes weak coupling.
  - Neglect radiative energy loss. The quark is not ultra-relativistic

$$\gamma v \lesssim \frac{1}{\alpha_s} \frac{m_{\rm D}}{T} \sim 6$$

  - Assumes a definite form for fluctuations
- Modeling
  - Input spectrum of charm and bottom quarks – from Cacciari *e.t. al*
  - Hadronize according to measured fragmentation functions
  - Electrons from charm and bottom semileptonic decays measured
  - Can not separated the charm and bottom contributions



Summary

1. Hard to reproduce the elliptic flow and suppression at the same time.
2. From the suppression pattern, we estimate that

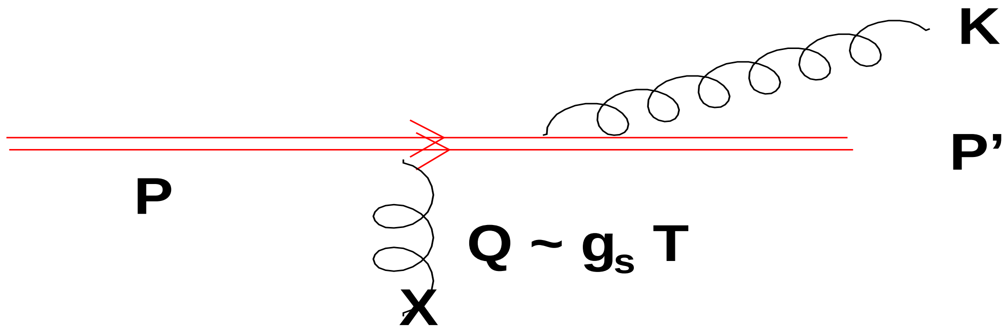
$$D \lesssim \frac{12}{2\pi T}$$

Order of magnitude consistency of transport coefficients

$$D \sim \frac{6}{2\pi T} \qquad \Leftrightarrow \qquad \frac{\eta}{s} = 2\frac{1}{4\pi}$$

Generalization to Relativistic Quarks

Radiation versus Collisions:

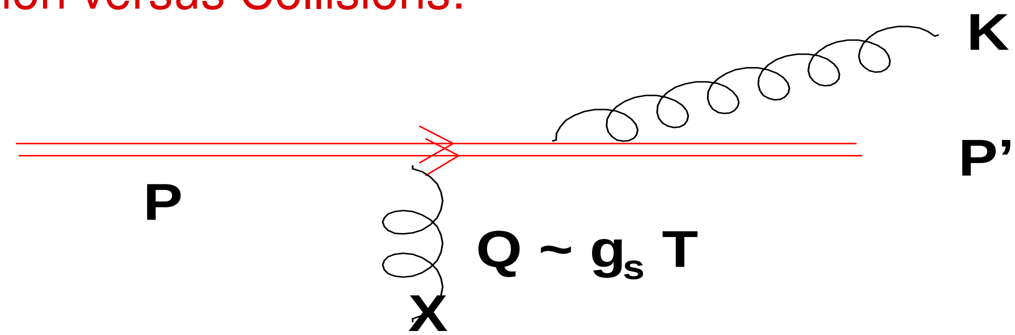


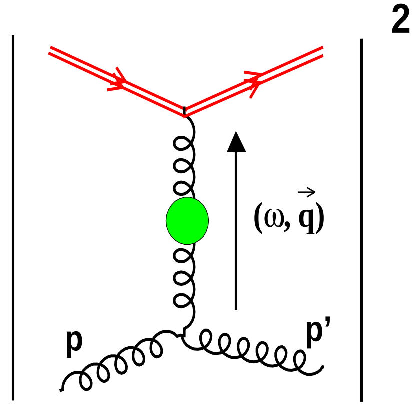
$$
\begin{aligned}
\text{Energy}: \quad k^0 &= p^0 - p'^0 = v(p - p') \\
\text{Momentum}: \quad k &= p - p' - q \\
\Longrightarrow \quad k &\leq \tfrac{qv}{1-v} \sim v(g_s T)/(1-v)
\end{aligned}
$$

- The Energy loss rate is:

$$
\begin{aligned}
\frac{dE}{dt} &\sim \underbrace{qv/(1-v)}_{\text{Bremmed Energy}} \times \underbrace{g^2 T}_{\text{Scattering Rate}} \times \underbrace{g_s^2}_{\text{Penalty}} \\
&\sim (\gamma v)\, g^5\, T^2
\end{aligned}
$$

Radiation versus Collisions:





- Collision E-Loss Rate: $\sim T(g^4 T)$
- Bremsstrahlung E-Loss Rate: $\sim (\gamma v) g^5 T^2$
- Thus collisions dominate loss until the heavy quark is ultra-relativistic:

$$(\gamma v) \sim \frac{1}{g}$$

- For QED radiation dominates for: $\gamma v \sim 750$. Could expect $\gamma v \sim 7$ for QCD.

I neglected radiation – not too good!

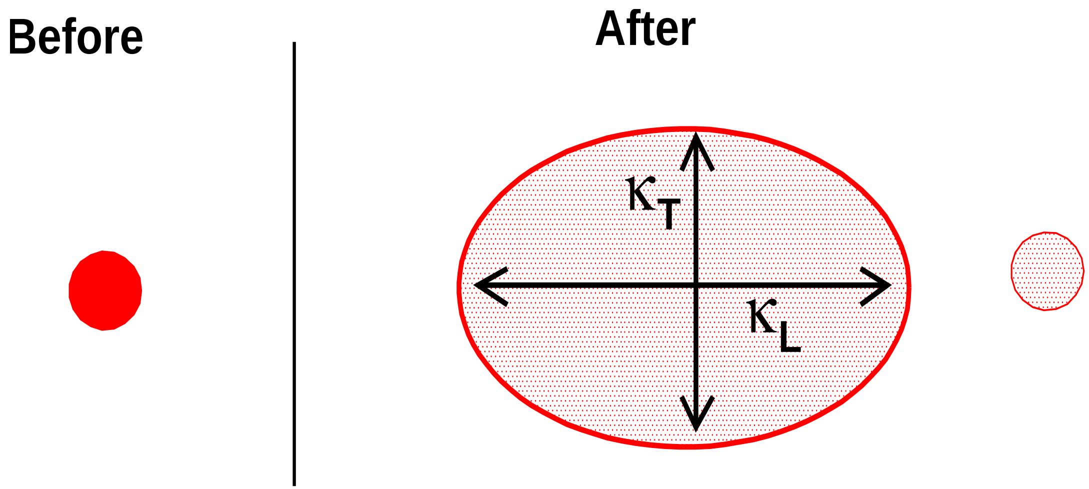
Relativistic Langevin:

- Write an equation of motion for the heavy quarks with drag and kicks.

$$\frac{dp^i}{dt} = -\eta_D(p)p^i + \xi^i$$

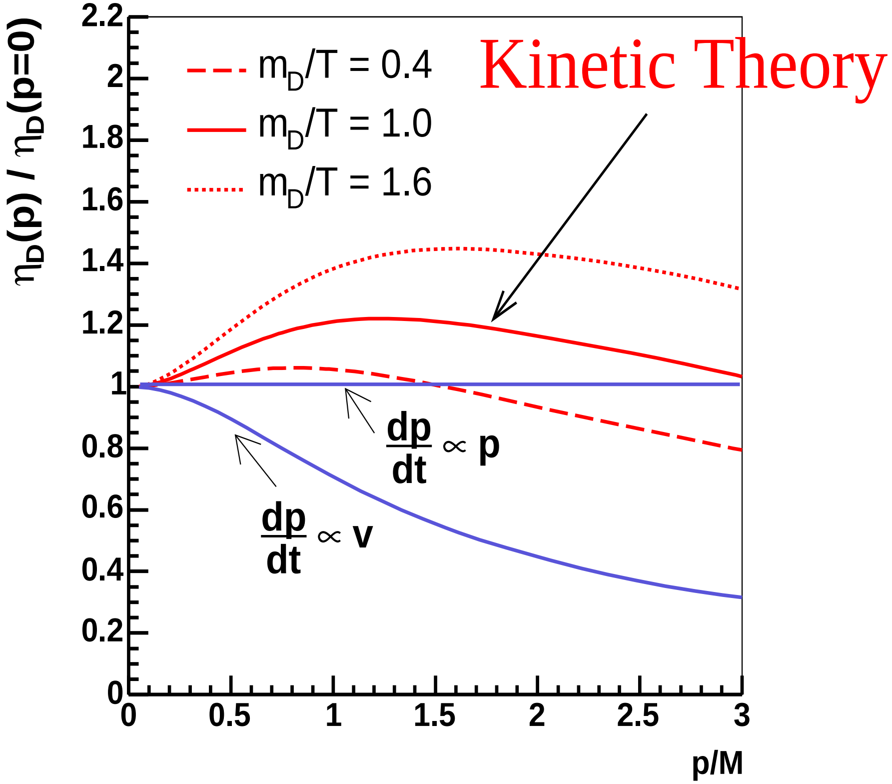
- We may replace the interaction by a random interaction

$$\langle \xi^i(t)\, \xi^j(t') \rangle = \delta_{tt'} \left[\kappa_L(p)\, \hat{p}^i \hat{p}^j + \kappa_T(p) \left(\delta^{ij} - \hat{p}^i \hat{p}^j \right) \right]$$



Go use kinetic theory to compute momentum dependence

Summary of drag coefficient as a function of momentum

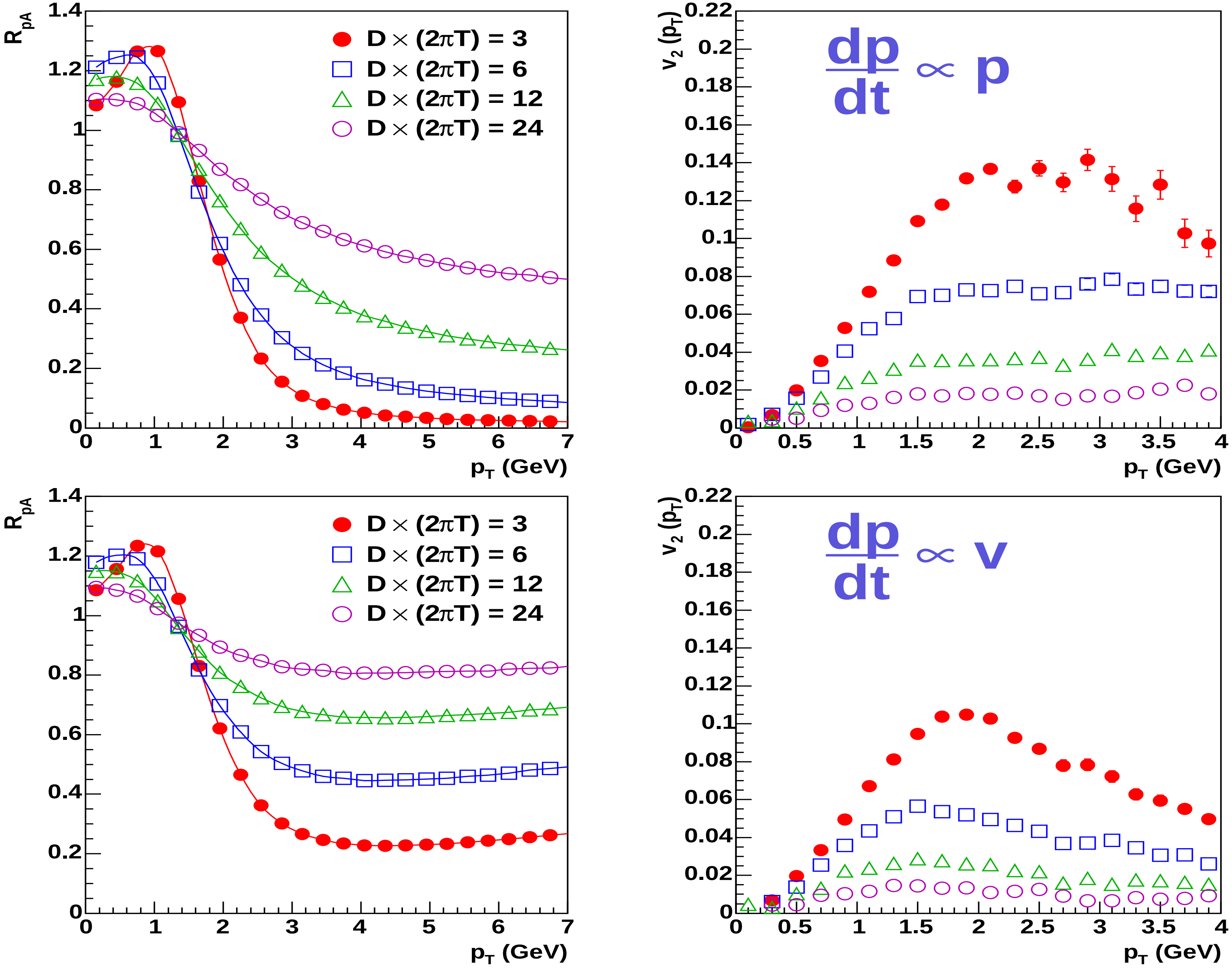


1. The relaxation time grows with p_T

$$\frac{dp}{dt} \propto v$$

2. The relaxation time is independent of p_T

$$\frac{dp}{dt} \propto p$$



Factor two uncertainty for D due to momentum dependence

Computing heavy quark diffusion coefficient

- Compute at weak coupling $\longrightarrow$ Kinetic Theory
- Lattice $\longrightarrow$ Hard
- Compute at strong coupling $\longrightarrow$ Model theories – AdS/CFT

Extrapolate to reality.

Giving the diffusion coefficient a rigorous definition

- Heavy Quarks are Quasi Classical

$$\lambda_{\text{de Broglie}} \sim \frac{\hbar}{\sqrt{MT}} \ll \frac{\hbar}{T}$$

- Compare the Langevin process to the microscopic theory

Langevin

$$\frac{dp}{dt} = -\eta_D p + \xi(t)$$

Microscopic Theory

$$\frac{dp}{dt} = \mathcal{F}(t, \mathbf{x}) = qE(t, \mathbf{x})$$

- Match the Langevin to the Microscopic Theory

Langevin

$$\kappa = \int dt \, \langle \xi(t) \, \xi(0) \rangle$$

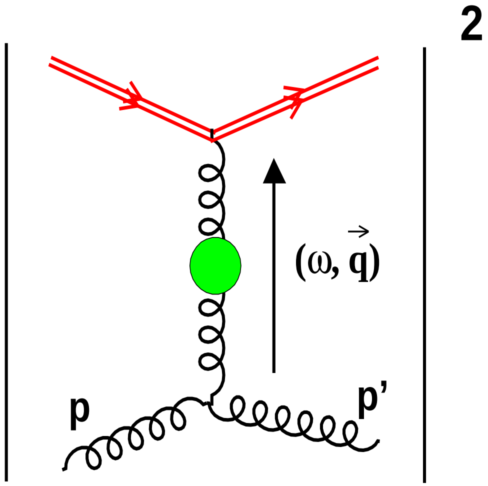
Microscopic Theory

$$\kappa = \int dt \, \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle_{HQ}$$

Diffusion Coefficient $\leftrightarrow$ Electric Field Correlator

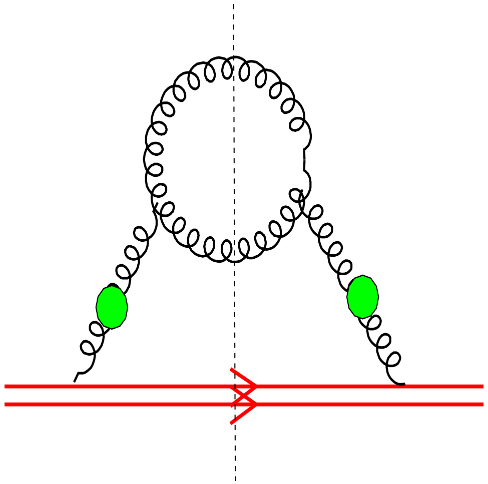
Computing κ – Kinetic Theory vs. Correlators

- κ is the mean squared momentum transfer per unit time:



$$\Longrightarrow \qquad \kappa = \int_{\mathbf{p},\mathbf{q}} \mathbf{q}^2 \; n(p)(1+n(p'))\; \left|M_{\text{glue}}\right|^2$$

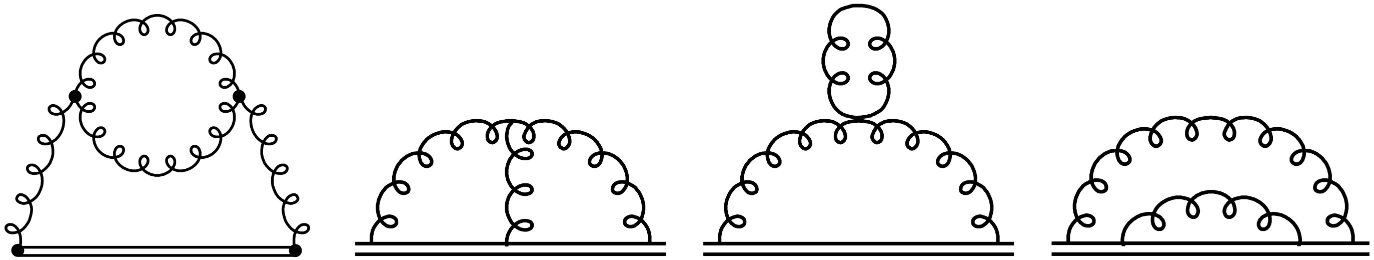
- κ is an Chromo-electric field correlator (+ Wilson Lines):



$\Longrightarrow$ The Same Thing

Beyond leading order (Guy D. Moore and Simon-Caron Huot)

(only transport coefficient known at NLO)



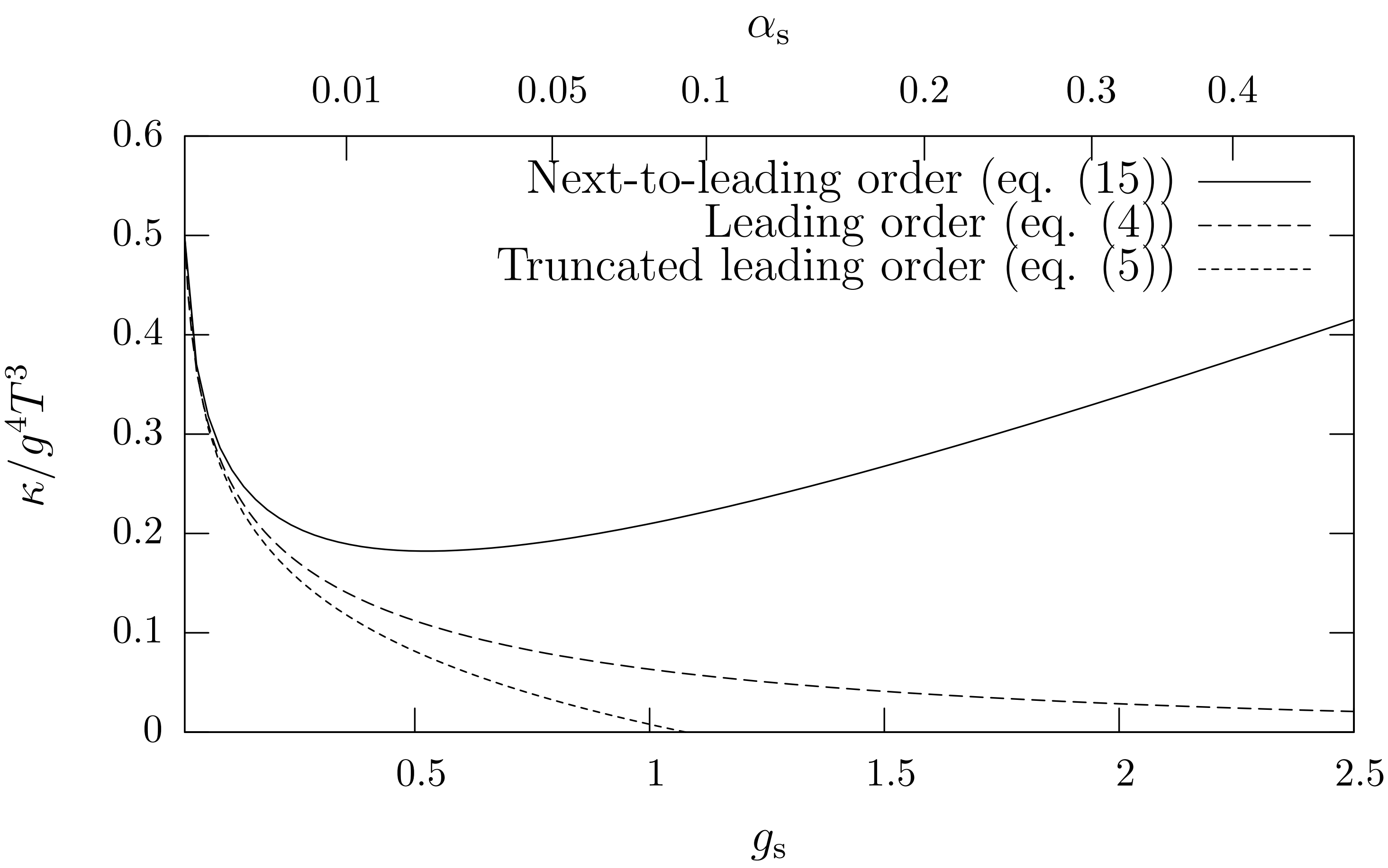
1. Perturbation theory in:

$$g_s \sim \frac{m_D}{T} \qquad \text{NOT} \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

2. Schematically:

$$\underbrace{\kappa}_{\text{diffusion rate}} = (g^4 T^3) \Big[\underbrace{C_0 \log\left(\frac{T}{m_D}\right) + C_1}_{\text{leading order}} + \underbrace{C_2 \frac{m_D}{T}}_{\text{NLO}} \Big]$$

(Guy D. Moore and Simon Carot-Huot)

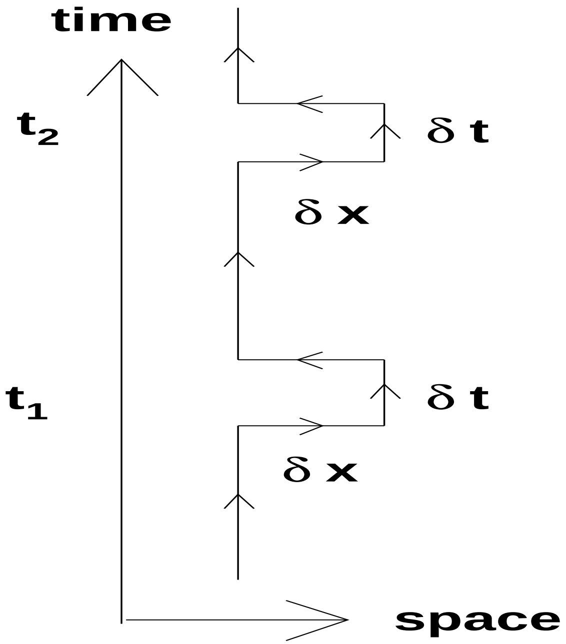


Perturbation theory fails for kinetics even for $T = M_Z$. More Resummation?

Computation in $\mathcal{N} = 4$ Super Yang Mills

(DT, J. Casalderrey; Herzog et al)

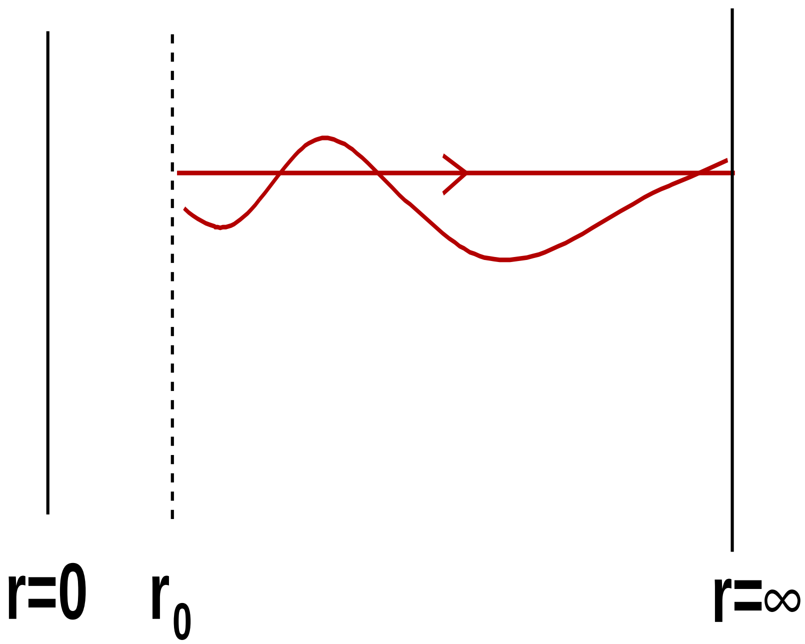
Boundary



$W[\delta x]$

$\Longrightarrow$

Bulk



$S_{\rm cl}[\delta x]$

$$\kappa = \sqrt{\lambda}\pi T^3 \qquad \lambda = g^2 N$$

QCD Guesses: Strong Coupling

- Strong Coupling: $\mathcal{N}=4$ SUSY. $\lambda \approx 5 \leftrightarrow 20$

$$D = \frac{2T^2}{\kappa} = \frac{1}{\sqrt{\lambda}}\frac{4}{2\pi T} \quad \longrightarrow \quad D \simeq \frac{1.0 \leftrightarrow 2.0}{2\pi T}$$

- Weak coupling (Aleski Vuorinnen)

2 gluons + 6 scalars + 8 fermions $\neq$ 2 gluons

$$\frac{D_{QCD}}{D_{SYM}} = \frac{6}{1+\frac{N_f}{2N_c}} \approx 4$$

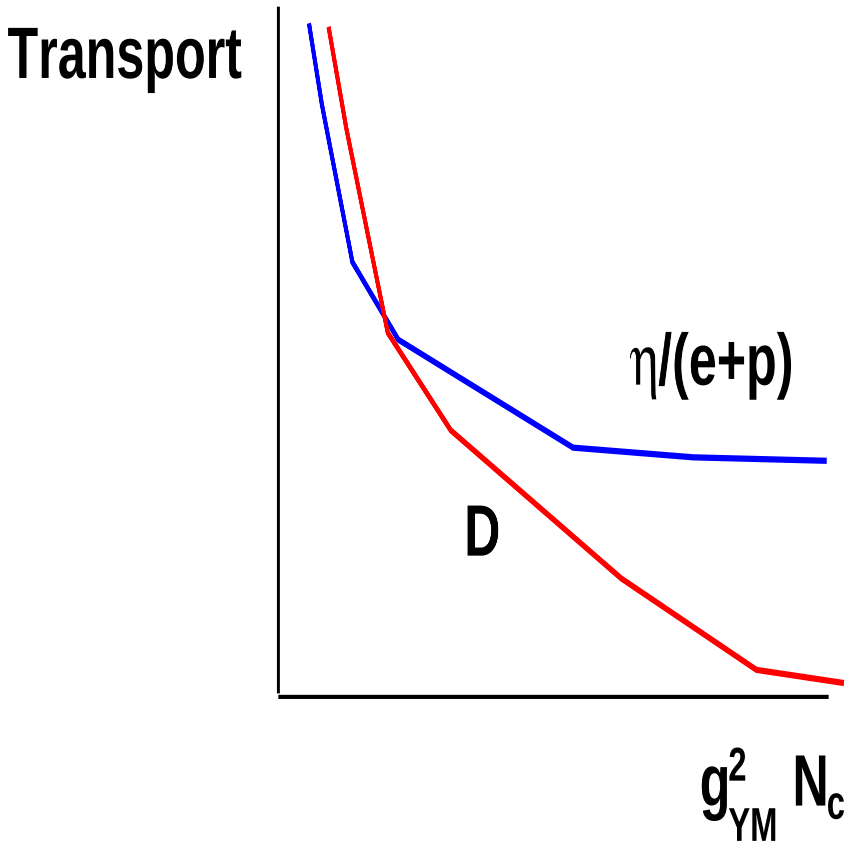
- Best guess for QCD from strong coupling

$$D \approx \frac{4.0 \leftrightarrow 8.0}{2\pi T}$$

Compare to weak coupling best guess $D \approx 6/(2\pi T)$

Heavy Quark Diffusion is Parametrically Small

$$D = \frac{1}{\sqrt{g_{YM}^2 N_c}} \frac{4}{2\pi T} \qquad\qquad \frac{\eta}{e+p} = \frac{1}{4\pi T}$$



Constraint On The Heavy Quark Mass

- To treat the heavy quark as a quasi-classical quasi-particle we need

$$\tau_R \gg \frac{\hbar}{T}$$

- Then we have

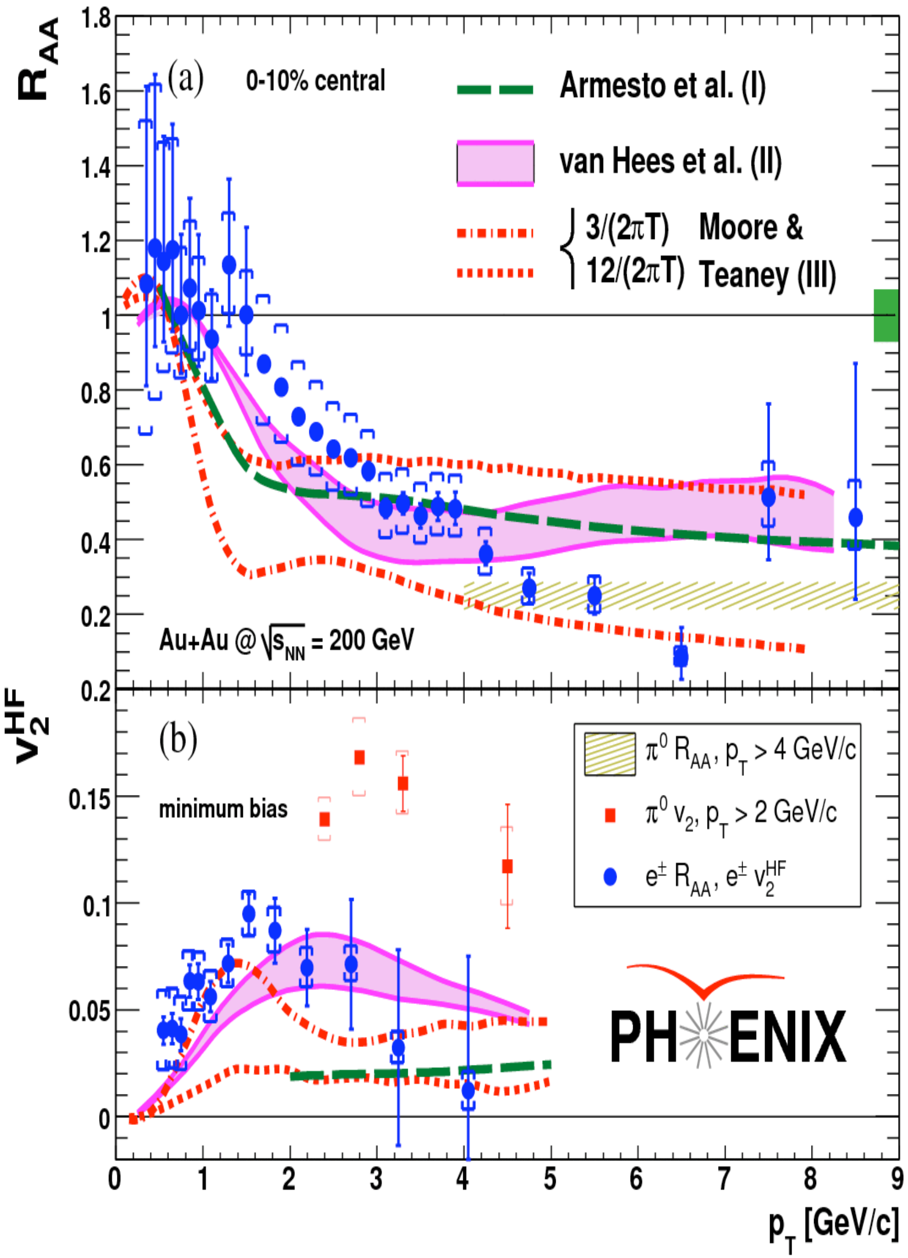
$$\tau_R \sim \frac{M}{T} D \qquad\qquad D = \frac{2}{\sqrt{\lambda}\pi T}$$

- This leads to a constraint on Mass/String Length

$$M \gg \frac{\pi T}{2}\sqrt{\lambda} \qquad\qquad L \gg r_o$$

- Substituting numbers we have

$$M \gg 1.7\text{GeV}\left(\frac{T}{0.250\,\text{GeV}}\right)\left(\frac{\alpha_{SYM} N}{1.5}\right)^{1/2}$$



Summary

- Perturbative QCD Estimates

$$D \approx \frac{3 \leftrightarrow 12}{2\pi T}$$

- Best guess for QCD from strong coupling

$$D \approx \frac{4.0 \leftrightarrow 8.0}{2\pi T}$$

Conclusions

- Order of mag. diffusion coefficient is consistent hydro transport coefficients.
- Getting to the data requires a significant modeling
  - b c crossing
  - Momentum dependence of kinetic coefficients
  - $\ldots$
- Computed diffusion coefficient both perturbatively and at strong coupling:

$$\underbrace{D \approx \frac{3 \leftrightarrow 12}{2\pi T}}_{\text{Weak Coupling}} \qquad \text{and} \qquad \underbrace{D \approx \frac{4 \leftrightarrow 8}{2\pi T}}_{\text{Strong Coupling}}$$